Cambridge Pre-U

MATHEMATICS **9794/02**

Paper 2 Pure Mathematics 2 **October/November 2020**

2 hours

0345364281

You must answer on the answer booklet/paper.

You will need: Answer booklet/paper
Graph paper
List of formulae (MF20)

INSTRUCTIONS

- Answer **all** questions.
- If you have been given an answer booklet, follow the instructions on the front cover of the answer booklet.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number on all the work you hand in.
- Do **not** use an erasable pen or correction fluid.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- At the end of the examination, fasten all your work together. Do **not** use staples, paper clips or glue.

INFORMATION

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [ ].

This syllabus is regulated for use in England, Wales and Northern Ireland as a Cambridge International Level 3 Pre-U Certificate.

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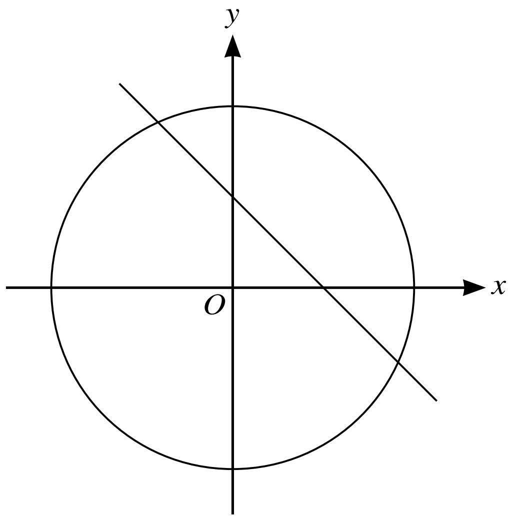
JC20 11_9794_02/FP

1 Find the sum of all the integers between 1 and 200 which are divisible by 7. [3]

2 Given that $\begin{pmatrix} a \\ b \\ 1 \end{pmatrix}$ is perpendicular to both $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$, find the values of a and b. [3]

3 The diagram shows the graphs of $x^2 + y^2 = 16$ and $y = 2 - x$.



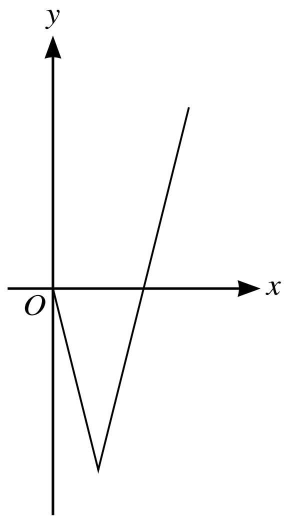
Find, in terms of $\sqrt{7}$, the exact coordinates of the points of intersection of the two graphs. [5]

4 **(a)** Express $6 \sin x + \sqrt{13} \cos x$ in the form $R \sin(x + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [4]

(b) Find the smallest positive value of x for which the maximum value of $6 \sin x + \sqrt{13} \cos x$ occurs. [2]

(c) Explain why the equation $6 \sin x + \sqrt{13} \cos x = -8$ has no solution. [1]

5 The diagram shows the graph of $y = \mathrm{g}(x)$, $0 \leqslant x \leqslant 3$, which consists of two straight line segments. One line segment joins the points $(0, 0)$ and $(1, -4)$, and the other line segment joins the points $(1, -4)$ and $(3, 4)$.



(a) Find $\int_0^3 \mathrm{g}(x)\,\mathrm{d}x$. [2]

(b) For $2 < x \leqslant 3$, let $\mathrm{h}(x) = \ln\big(\mathrm{g}(x)\big)$. Determine $\mathrm{h}'(2.5)$. [3]

(c) Describe fully the transformation which, when applied to the graph of $y = \mathrm{g}(x)$, will produce the graph of $y = \mathrm{g}(x - 2) + 1$. [1]

6 Find the coefficient of $\frac{1}{x^2}$ in the expansion of $(x + 1)^3 \left(\frac{1}{x} - 3x\right)^6$. [6]

7 **(a)** By sketching the graphs of $y = x^2$ and $y = \sin x$ on the same diagram, show that the equation $\sin x = x^2$ has exactly one positive root α in the interval $-\frac{1}{2}\pi \leqslant x \leqslant \frac{1}{2}\pi$. [2]

(b) Use the Newton-Raphson iteration method with $\mathrm{f}(x) = \sin x - x^2$ and $x_0 = 1$ to find the values of x_1, x_2 and x_3, correct to 3 significant figures. [3]

(c) Use a change of sign method to verify that $x_3 = \alpha$, correct to 3 significant figures. [2]

8 A line has equation $a(a^2 + 1)y = 3a - x$, where a is positive. This line intersects the x-axis at A and the y-axis at B. The origin is O.

(a) Show that the area of triangle AOB is $\frac{9a}{2(a^2 + 1)}$. [3]

(b) Find the maximum possible area of triangle AOB as a varies. [7]

9 **(a)** Given that $1 - \mathrm{i}$ is a root of the equation $2z^3 - 7z^2 + 10z - 6 = 0$, find the other two roots. [5]

(b) Given that $z = 1 - \mathrm{i}$, write down z^* and find zz^*.

Plot points P, Q and R representing the numbers z, z^* and zz^* respectively on an Argand diagram. Determine the angle PRQ. [5]

10 **(a)** Prove that $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$. [4]

(b) Prove also that $4\cos\theta\cos\left(\frac{1}{3}\pi - \theta\right)\cos\left(\frac{1}{3}\pi + \theta\right) = 4\cos^3\theta - 3\cos\theta$. [5]

11 **(a)** Show that $\dfrac{1}{(u+1)^2} + \dfrac{1}{(u+1)} + \dfrac{1}{(u-1)^2} - \dfrac{1}{(u-1)} = \dfrac{k}{(u+1)^2(u-1)^2}$, where k is a constant to be determined. [5]

(b) Using the substitution $u^2 = \mathrm{e}^x + 1$, show that

$$\int \frac{1}{\mathrm{e}^x\sqrt{\mathrm{e}^x + 1}}\,\mathrm{d}x = \int \frac{2}{(u+1)^2(u-1)^2}\,\mathrm{d}u.$$ [4]

(c) Find the exact value of $\displaystyle\int_{\ln 3}^{\ln 8} \frac{1}{\mathrm{e}^x\sqrt{\mathrm{e}^x + 1}}\,\mathrm{d}x$, giving your answer in the form $a + b\ln c$, where a, b and c are rational. [5]